

SPIRAL PROGRESSION in the K + 12 MATHEMATICS CURRICULUM

Soledad A. Ulep

University of the Philippines

National Institute for Science and Mathematics Education Development
(UP NISMED)



Objective of the Presentation

To enable the participants to understand the underlying principles and nuances of the spiral progression model in teaching Mathematics



Coverage of the Presentation

- ▶ K + 12 Mathematics Curriculum Framework
- ▶ Spiral Progression Approach and its Benefits
- ▶ Example of a Lesson that Teaches Mathematics through Problem Solving
- ▶ Examples of Using the Spiral Progression Approach



The K + 12 Mathematics Curriculum Framework



Goals of Mathematics in K to 12

- ▶ Critical thinking
- ▶ Problem solving



How to Achieve these Goals

- ▶ Content
- ▶ Skills and process
- ▶ Values and attitudes
- ▶ Tools
- ▶ Context



Content

- ▶ Numbers and Number Sense
- ▶ Measurement
- ▶ Geometry
- ▶ Patterns and Algebra
- ▶ Statistics and Probability



Skills and Processes

- ▶ Knowing and Understanding
- ▶ Estimating
- ▶ Computing and Solving
- ▶ Visualizing and Modeling
- ▶ Representing and Communicating
- ▶ Conjecturing, Reasoning, Proving and Decision-Making
- ▶ Applying and Connecting



Values and Attitudes

- ▶ Accuracy
- ▶ Creativity
- ▶ Objectivity
- ▶ Perseverance
- ▶ Productivity



Tools

- ▶ Manipulative objects
- ▶ Measuring devices
- ▶ Calculators and Computers
- ▶ Smartphones and Tablet PCs
- ▶ Internet



Contexts

- ▶ Beliefs
- ▶ Environment
- ▶ Language
- ▶ Culture
- ▶ Learner's prior knowledge and experiences



Underlying Learning Principles and Theories

- ▶ Experiential and Situated Learning
- ▶ Reflective Learning
- ▶ Constructivism
- ▶ Cooperative Learning
- ▶ Discovery and Inquiry-based Learning



Spiral Curriculum

- ▶ Builds upon previously learned knowledge
- ▶ Deepens understanding of topics



Features

- ▶ Learners revisit a topic.
- ▶ The complexity of the topic increases with each revisit.
- ▶ New knowledge being developed in each revisit has a relationship with previously learned knowledge.



Benefits of Using Spiral Progression

- ▶ It helps reinforce learning.
- ▶ It allows a logical progression from simple to complex ideas.
- ▶ It encourages students to apply their previous learning to later topics and new situations.
- ▶ It helps learners appreciate the connections among the different content strands.



Problem Solving

***Problem solving should not only be a goal of learning mathematics.
Problem solving should also be a means of learning mathematics.***



Problem

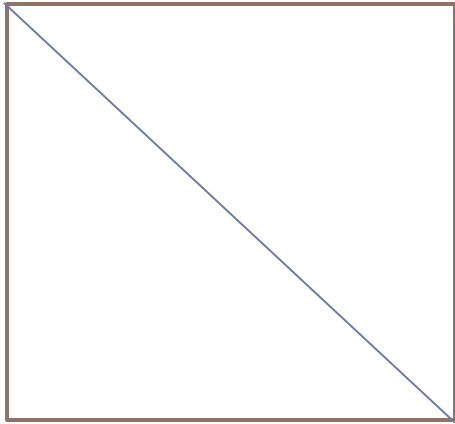
(Grade 1)

Karen has a piece of square paper. She wants to share one-half of the paper to Jun.

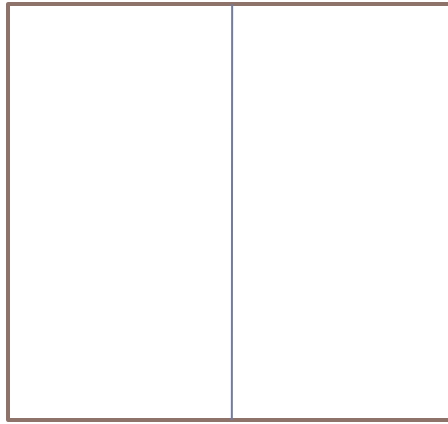
If you were Karen, how would you divide the paper?



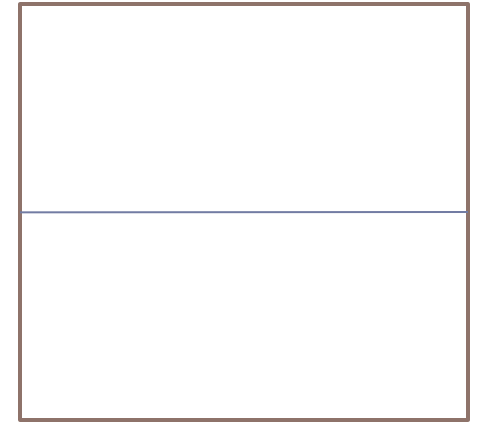
Different solutions:



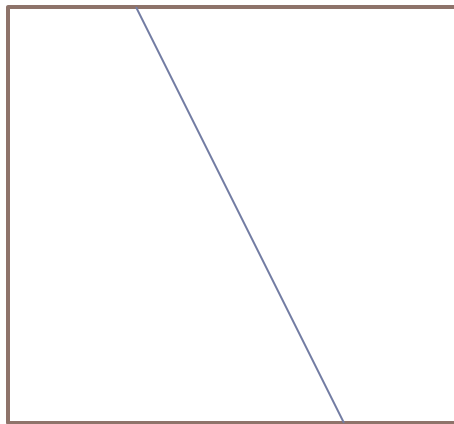
Solution 1



Solution 2



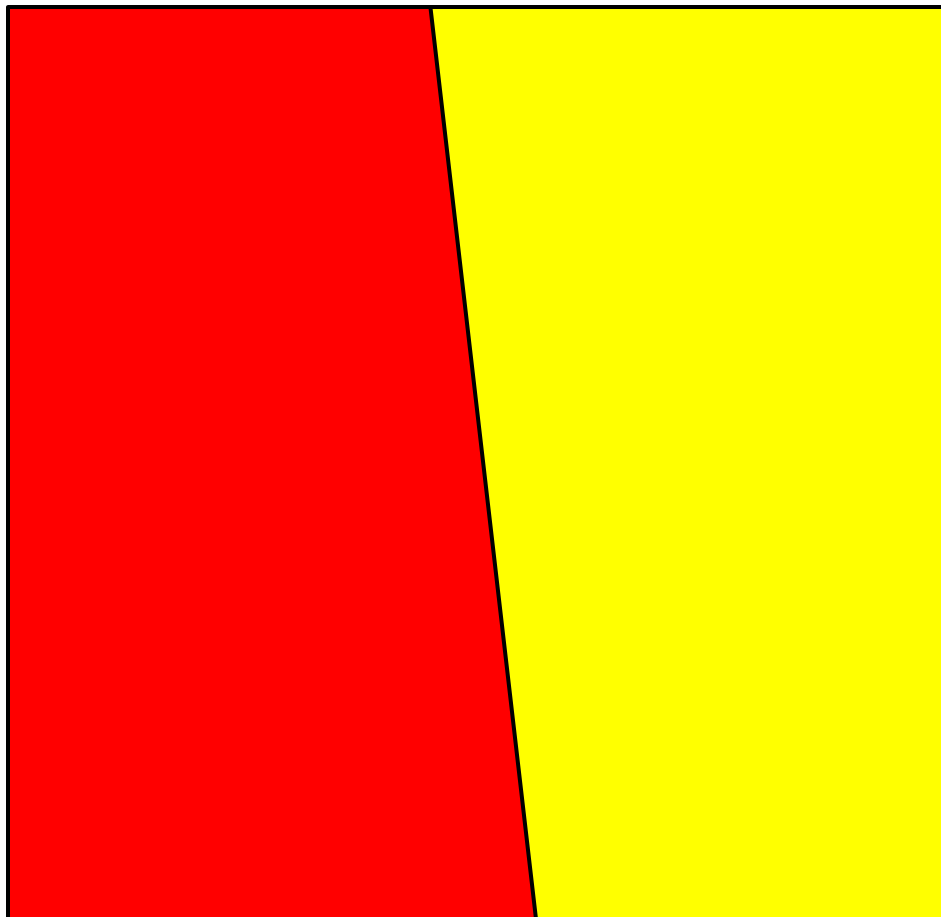
Solution 3



Solution 4

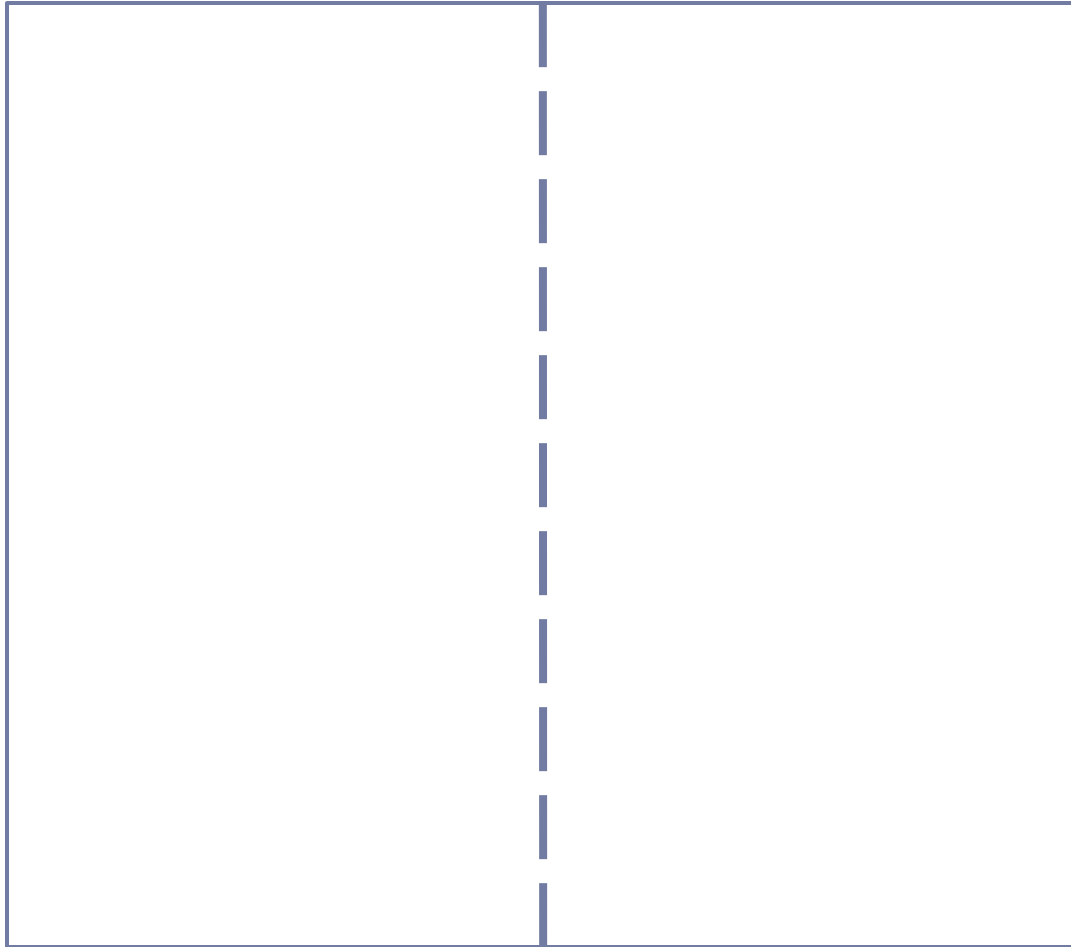


Verification of Solution 4

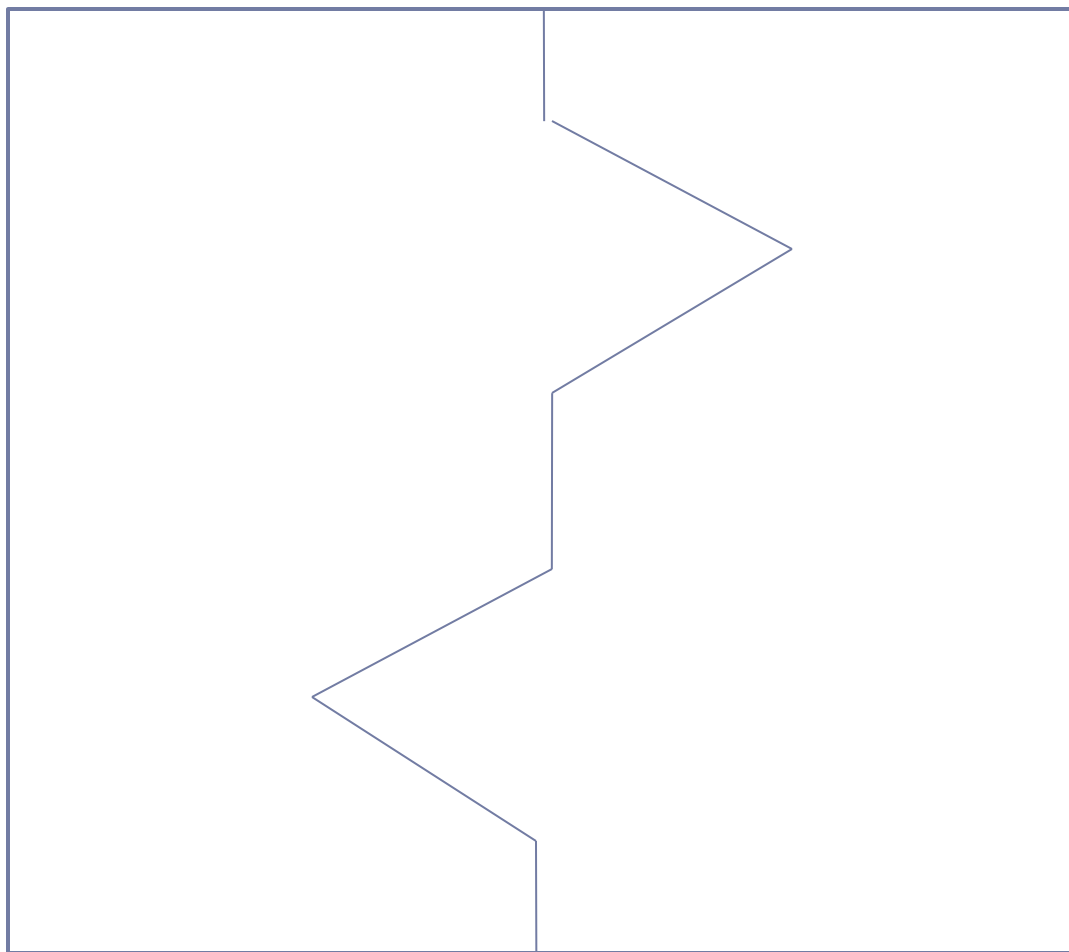


Other ways :

Start with folding a sheet of square paper this way



Example:



Area Measurement in the K +12 Curriculum



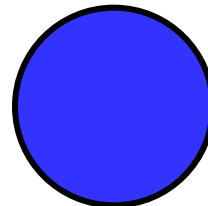
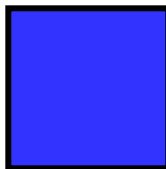
Area Measurement

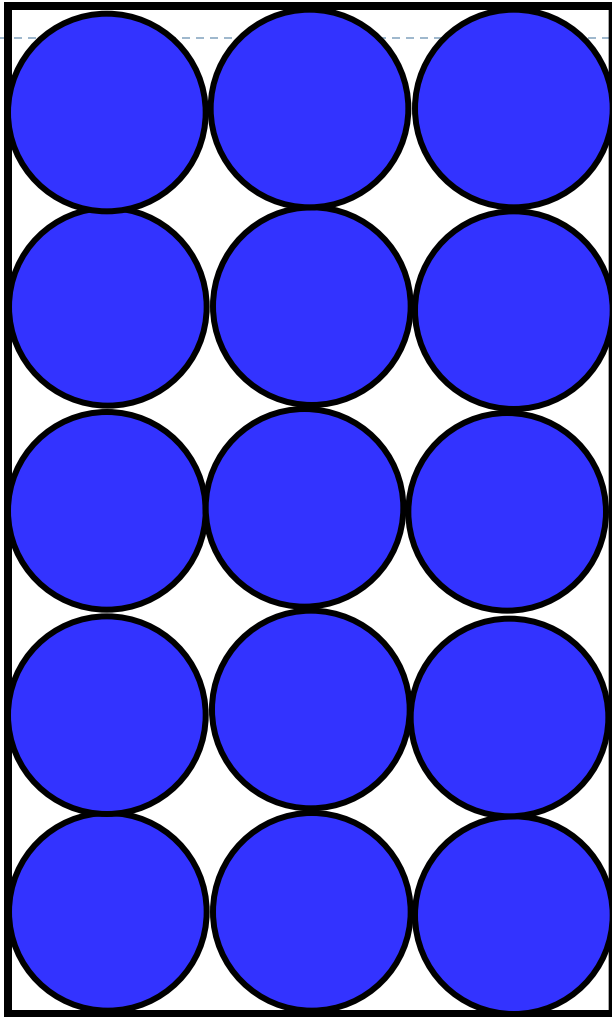
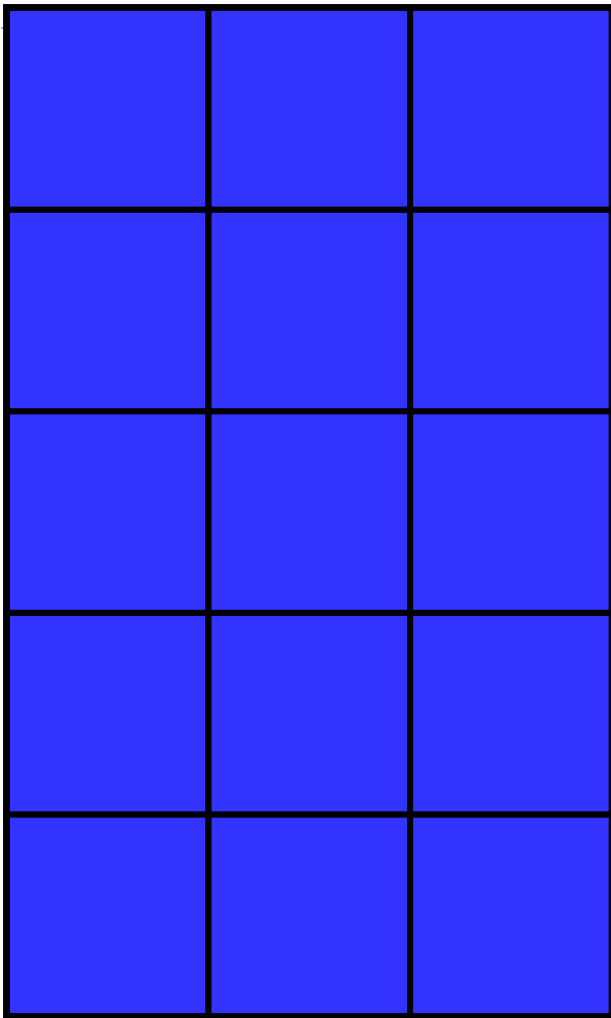


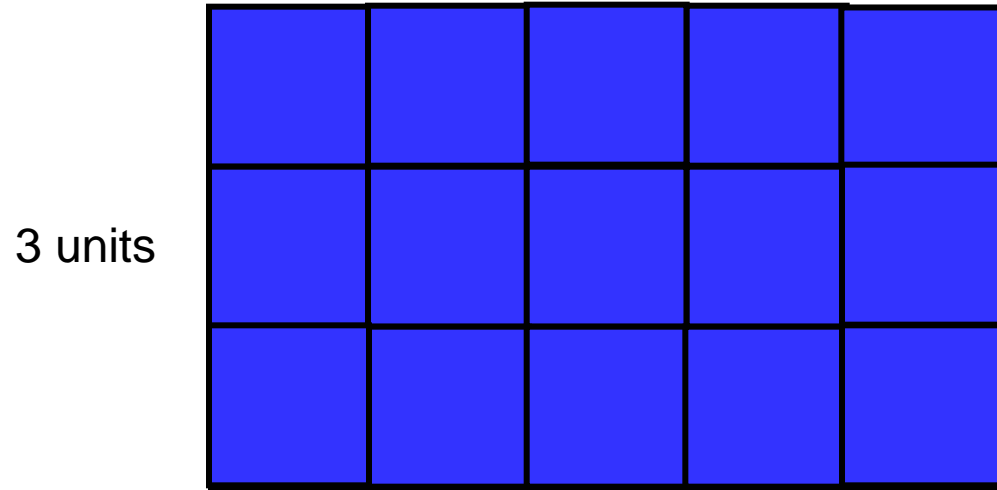
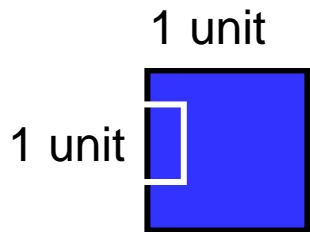
Grade 2



Area Measurement





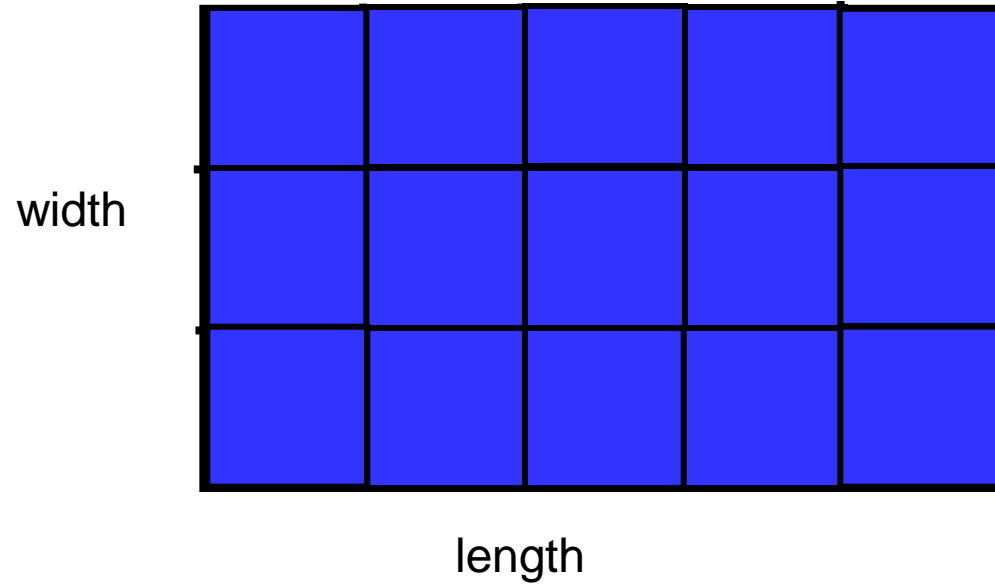


5 units

$$\begin{aligned} \text{Area} &= 3 \times 5 \\ &= 15 \text{ square units} \end{aligned}$$

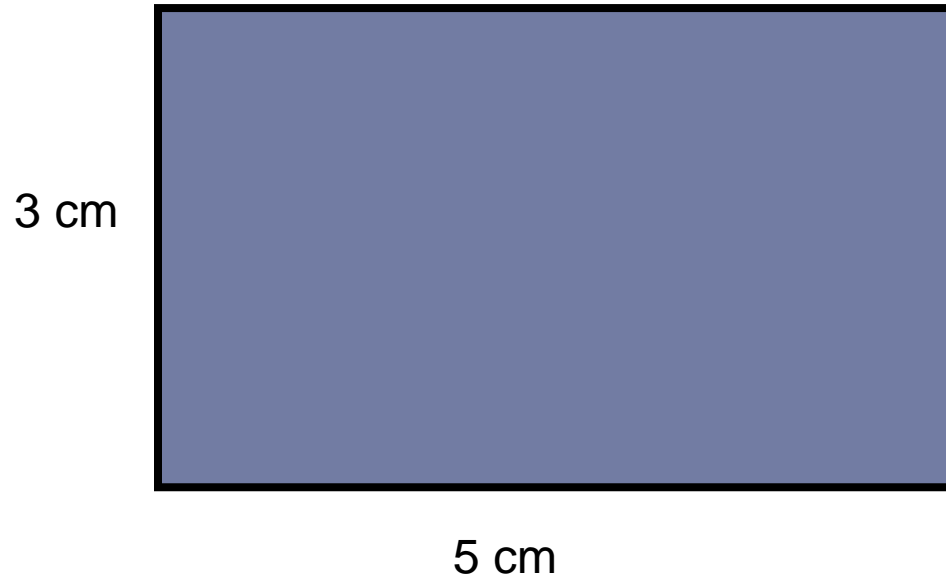
Grade 3





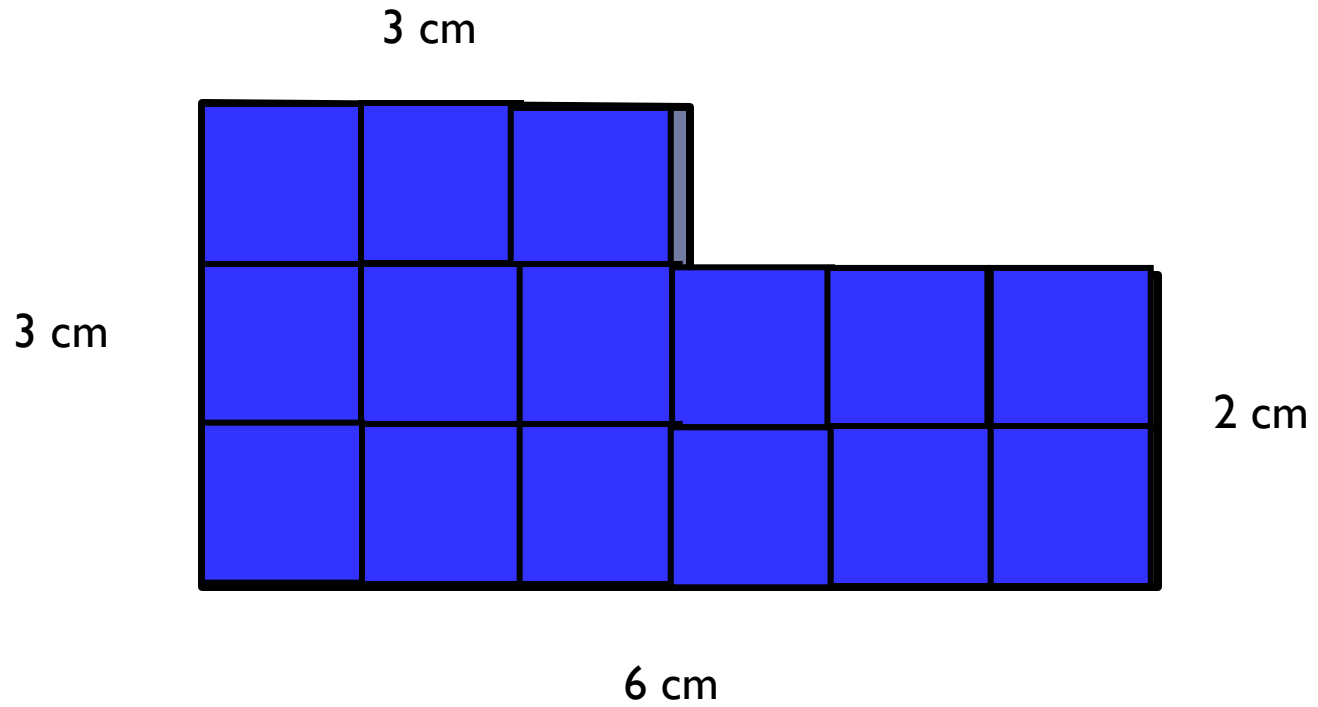
$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ &= l \times w\end{aligned}$$





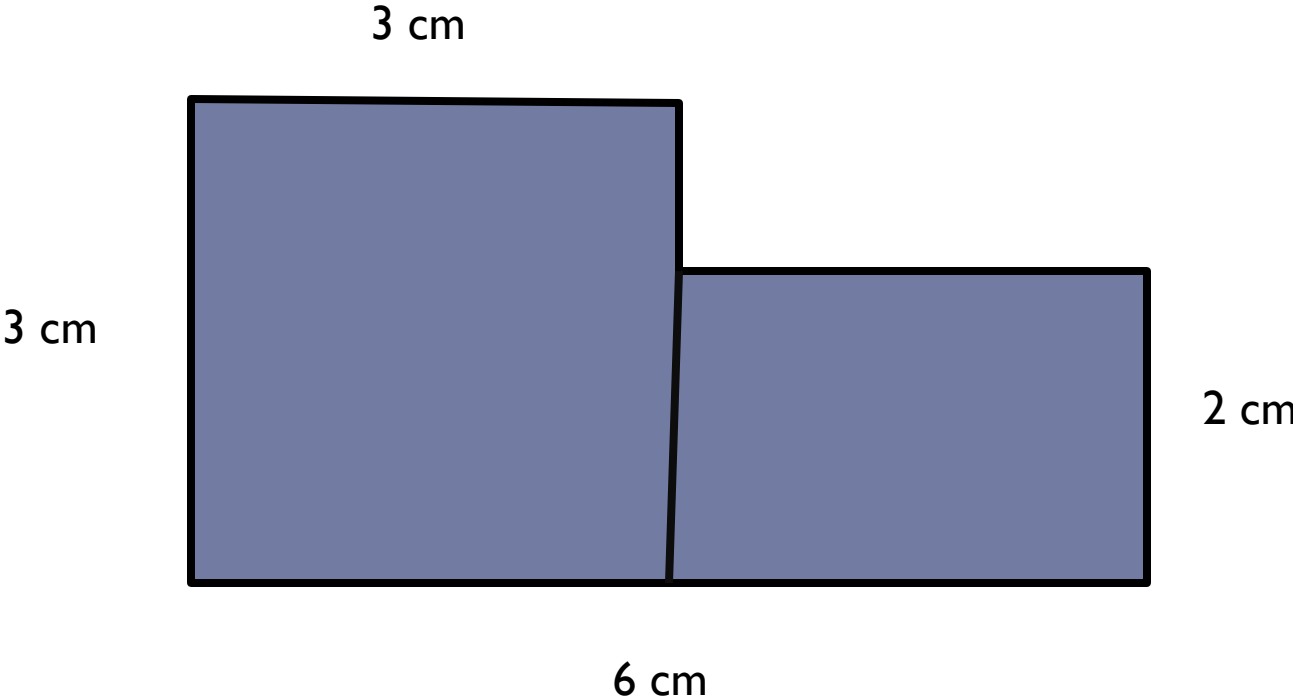
$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ &= l \times w \\ &= 3 \times 5 \\ &= 15 \text{ square cm}\end{aligned}$$





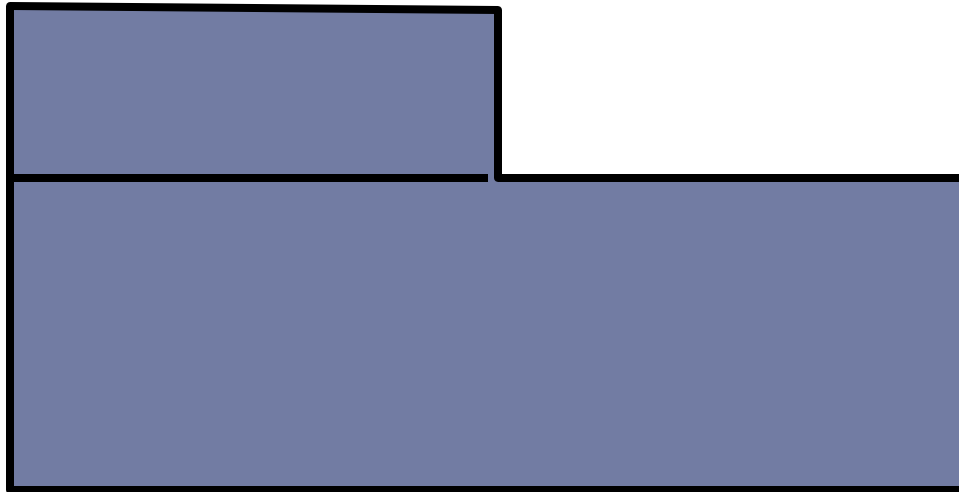
Grade 4







3 cm



3 cm

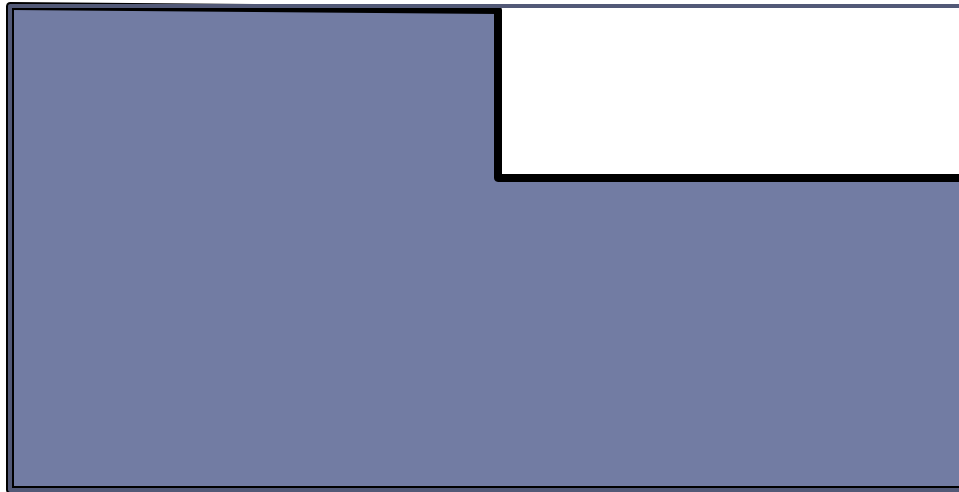
2 cm

6 cm





3 cm

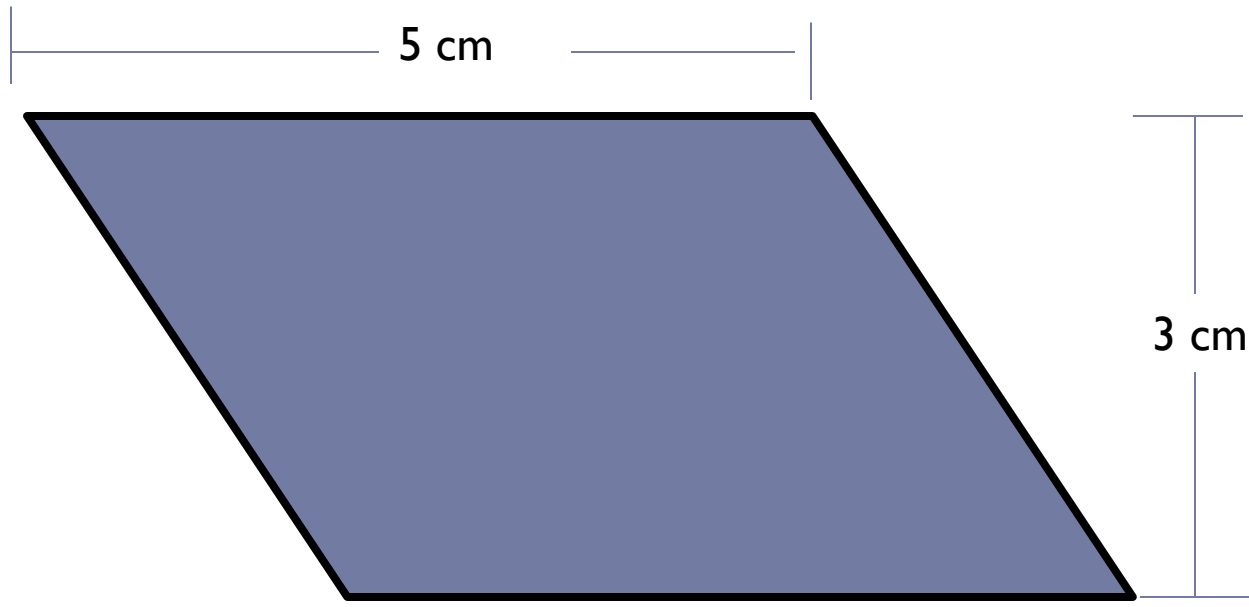


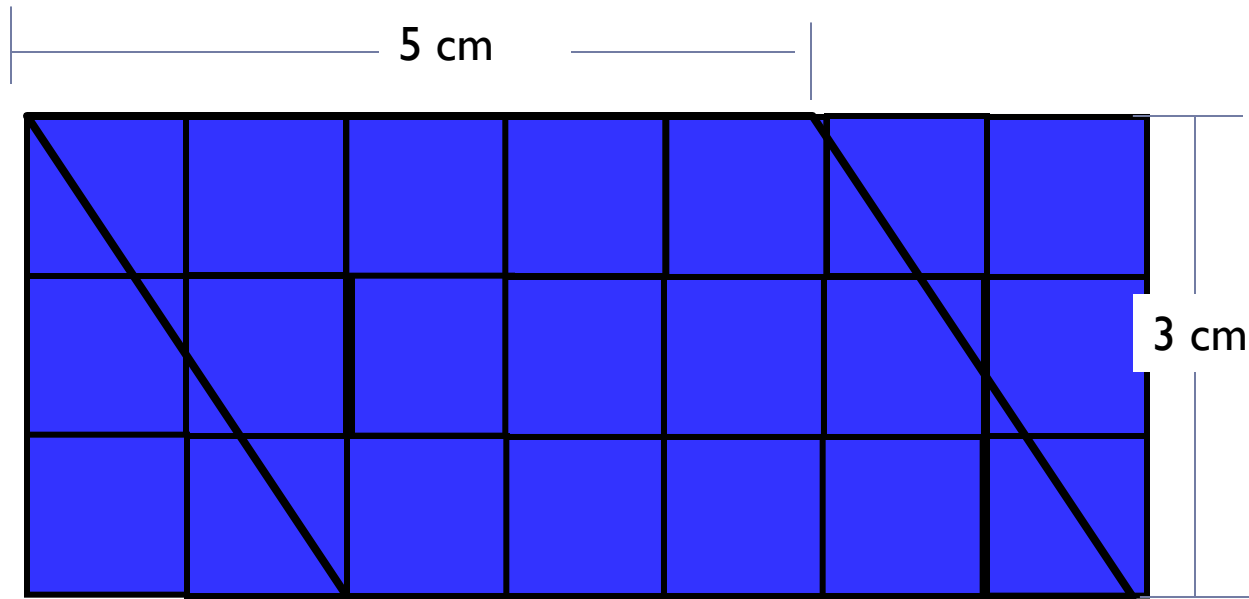
3 cm

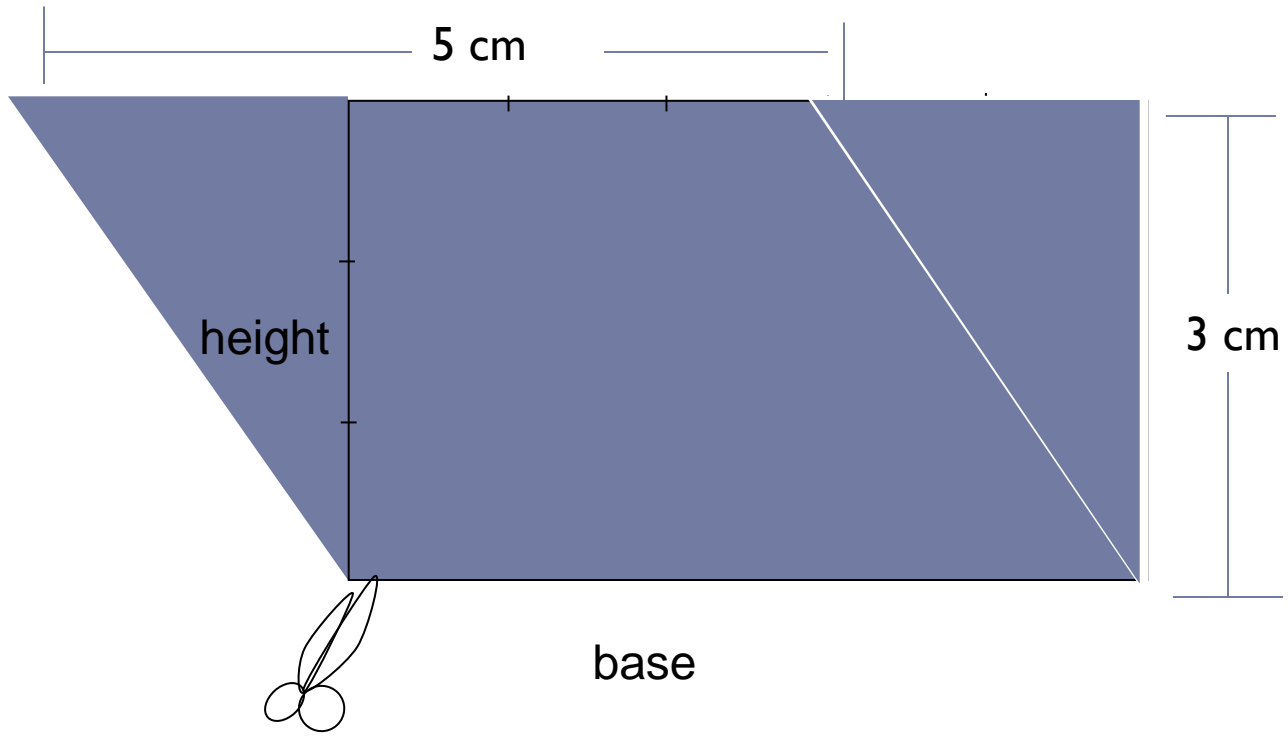
2 cm

6 cm









$$\begin{aligned} \text{Area of parallelogram} &= \text{base} \times \text{height} \\ &= bh \end{aligned}$$

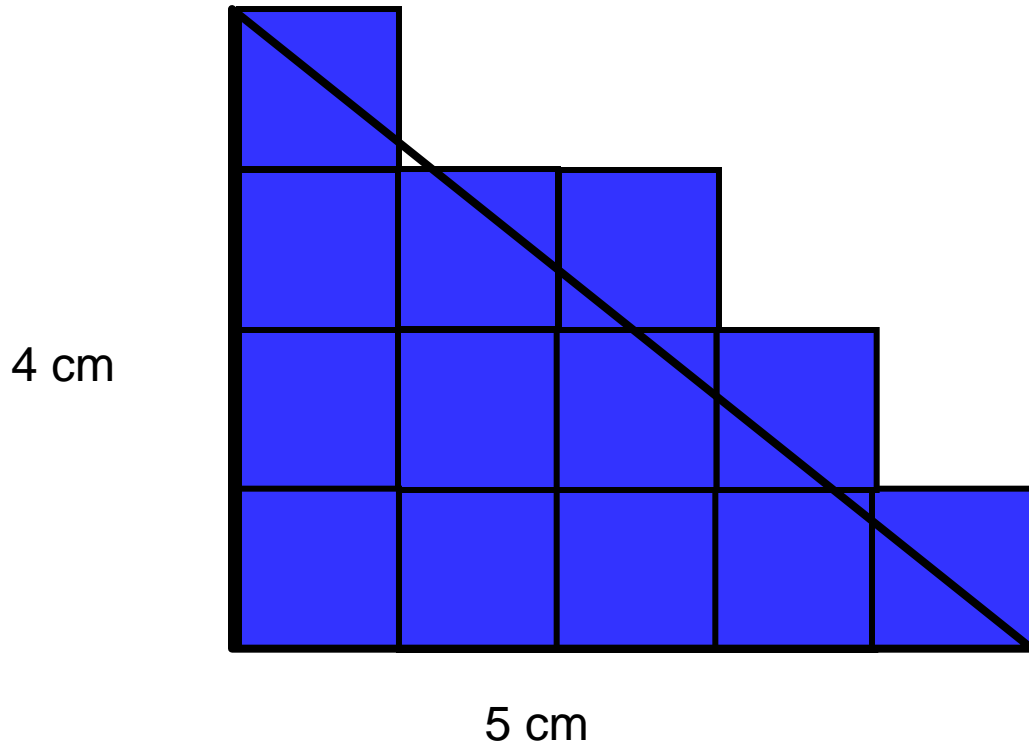


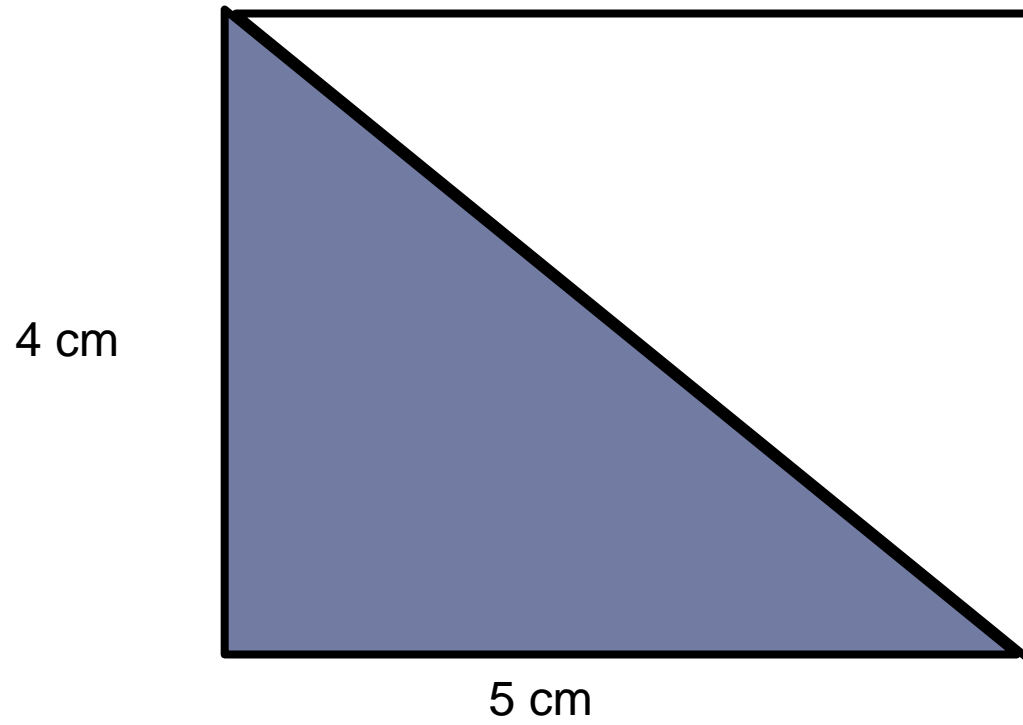


4 cm

5 cm

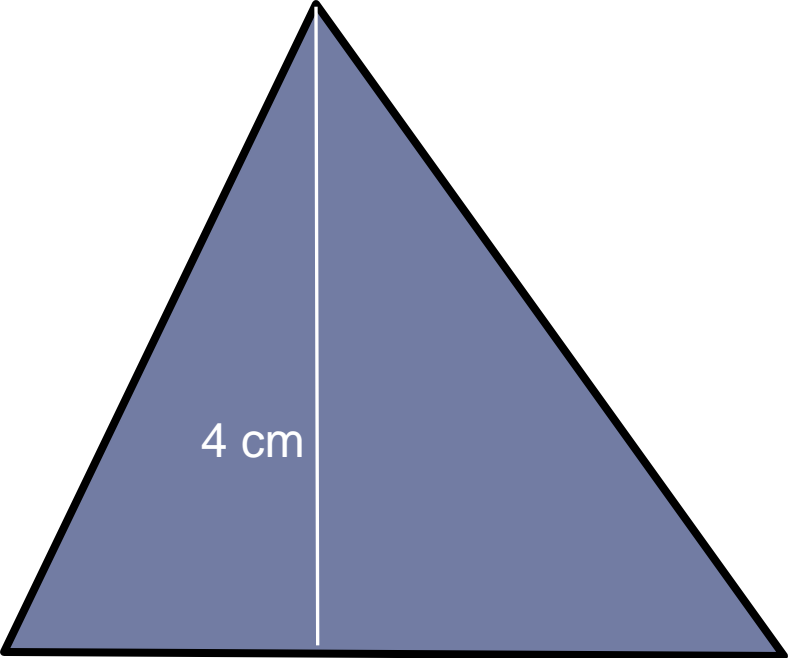






$$\text{Area} = \frac{bh}{2}$$

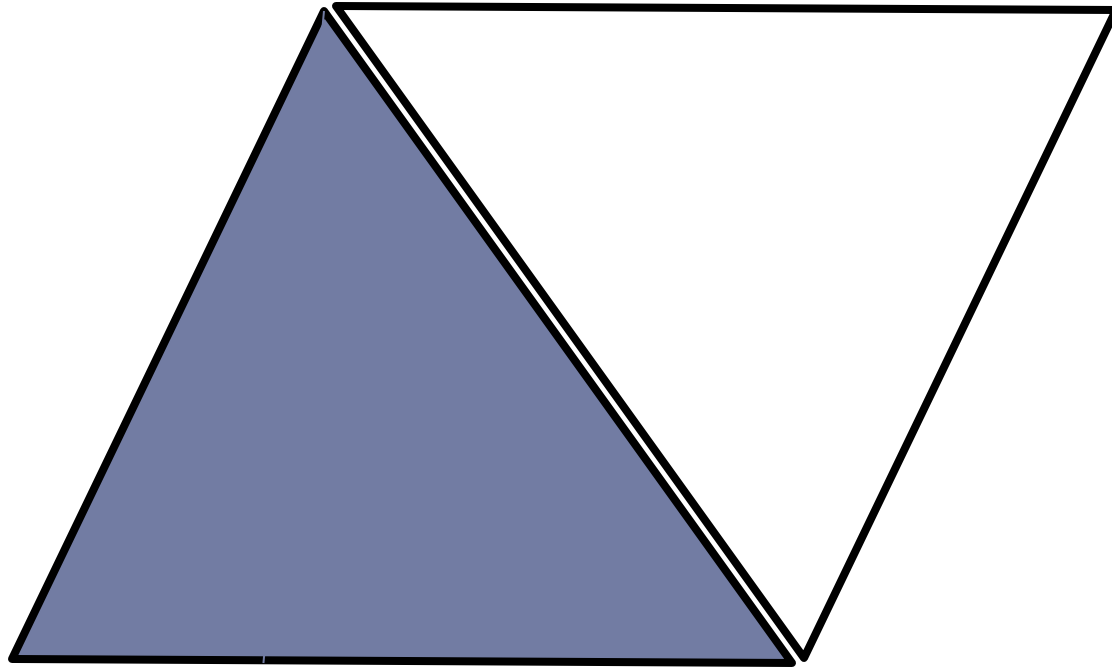




5 cm

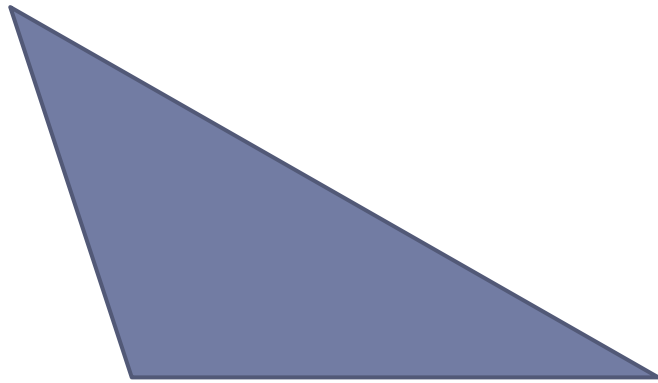
4 cm

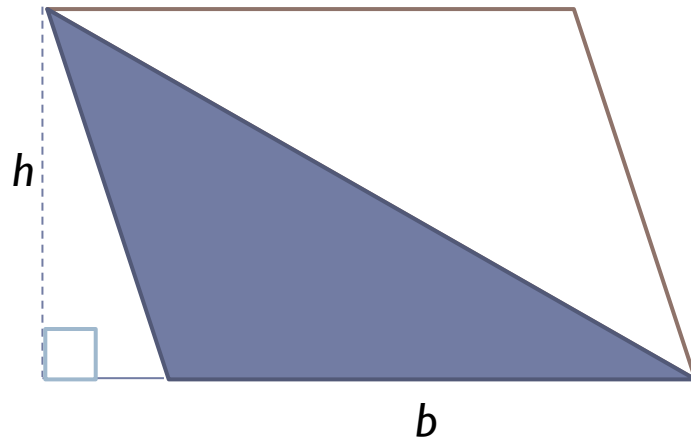




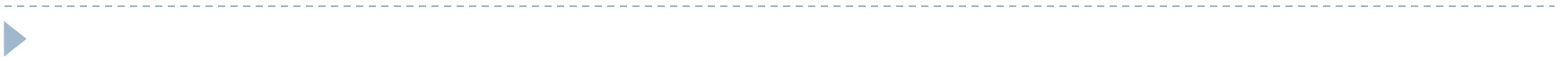
$$\text{Area} = \frac{bh}{2}$$

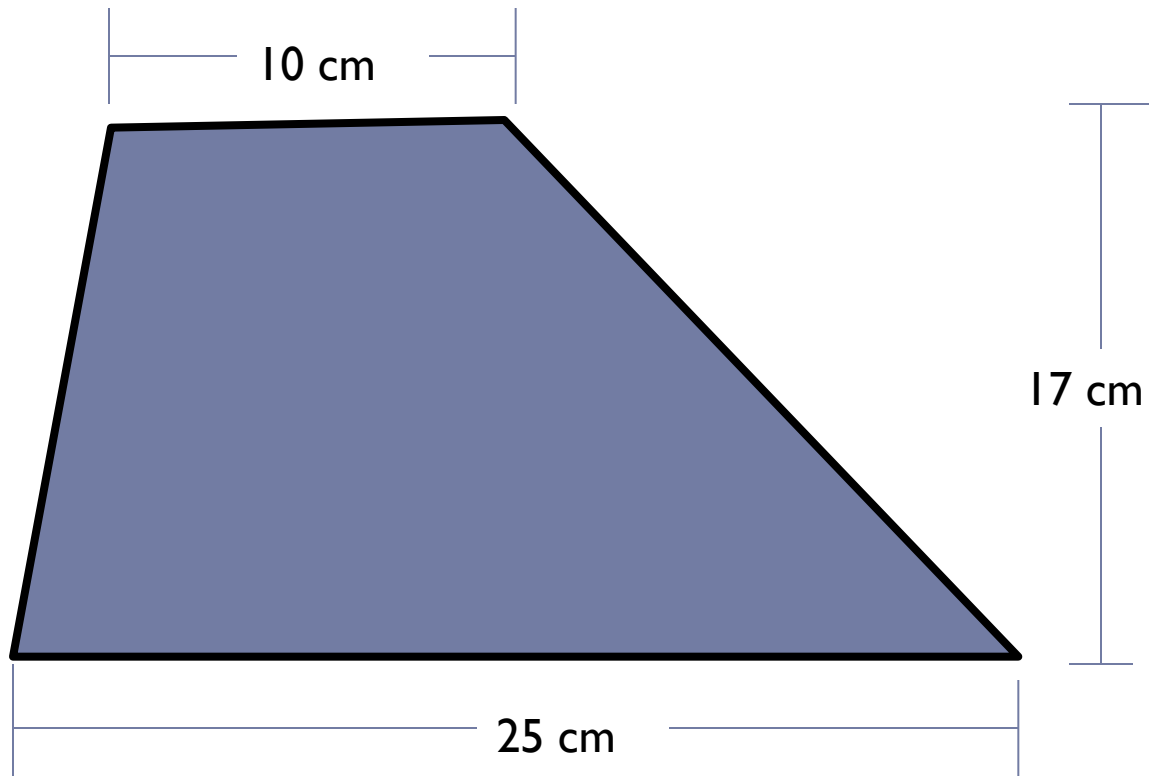


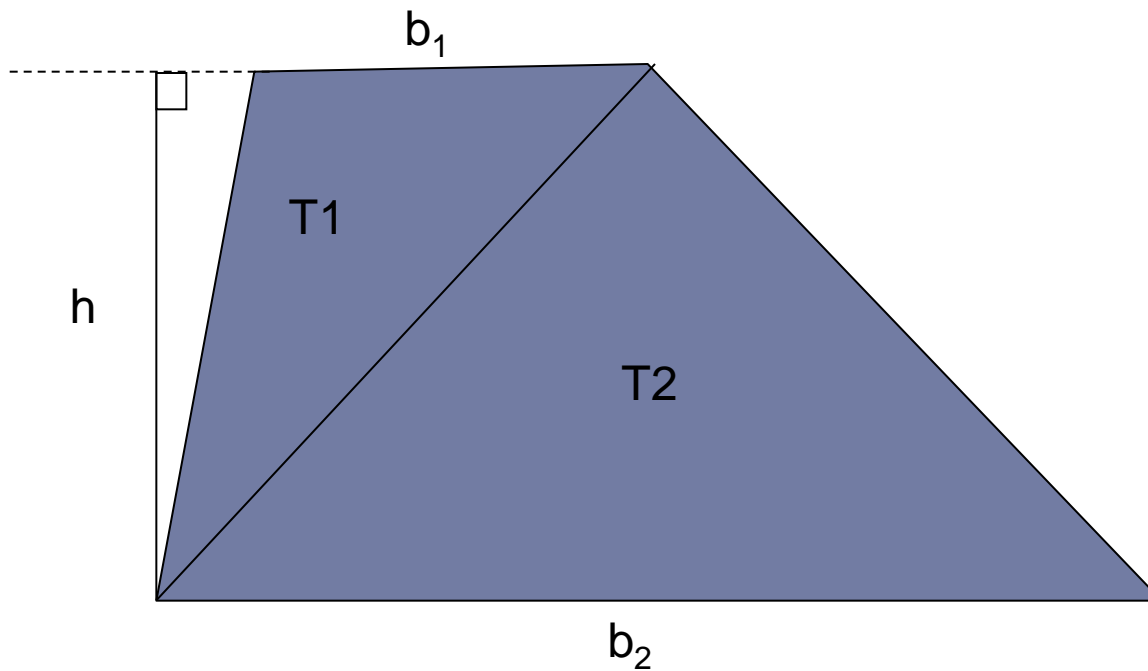




$$\text{Area} = \frac{bh}{2}$$







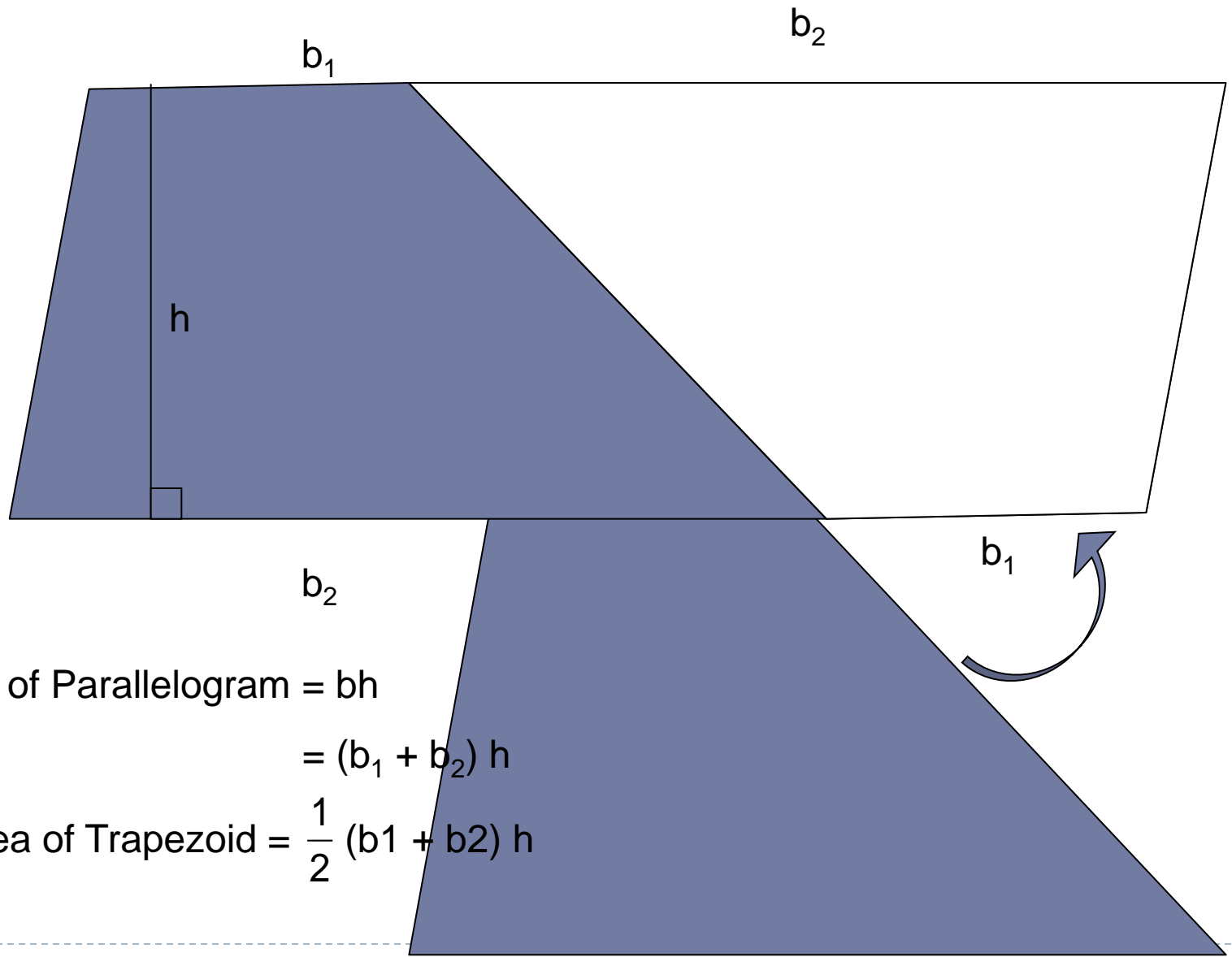
Area of Trapezoid = Area of T1 + Area of T2

$$= \frac{b_1 h}{2} + \frac{b_2 h}{2}$$

$$= \frac{h(b_1 + b_2)}{2}$$

$$= \frac{1}{2}(b_1 + b_2)h$$

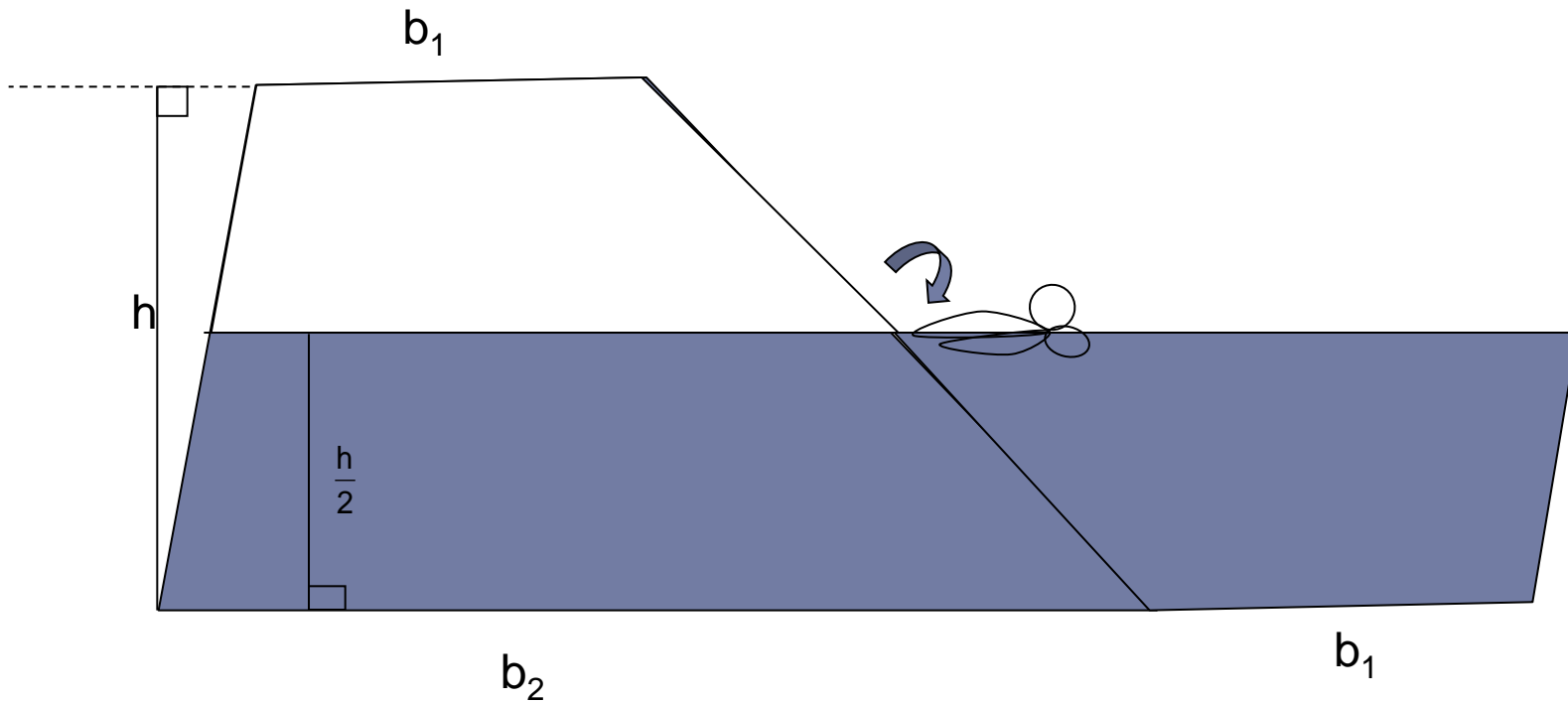




Area of Parallelogram = bh

$$= (b_1 + b_2) h$$

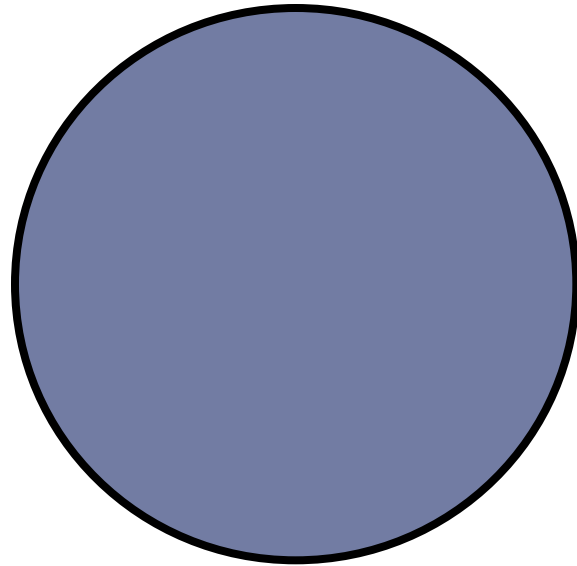
$$\text{Area of Trapezoid} = \frac{1}{2} (b_1 + b_2) h$$



$$\begin{aligned} \text{Area of Parallelogram} &= bh \\ &= (b_1 + b_2) \frac{h}{2} \end{aligned}$$

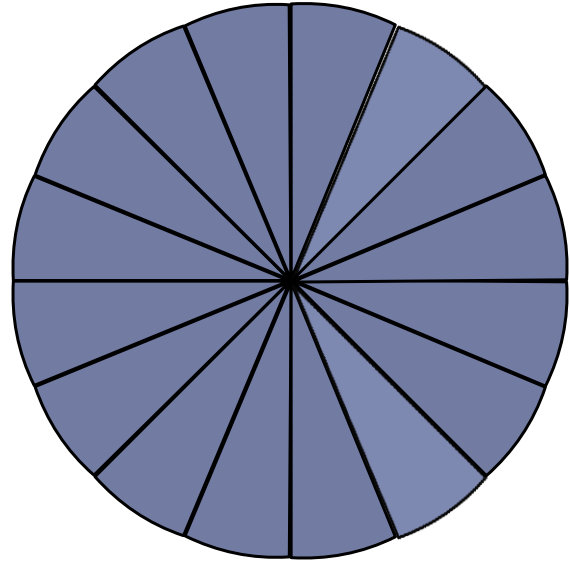
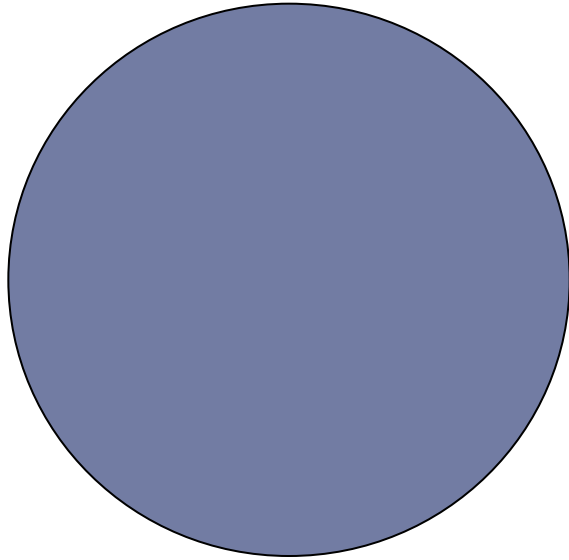
$$\text{Area of Trapezoid} = \frac{1}{2} (b_1 + b_2) h$$

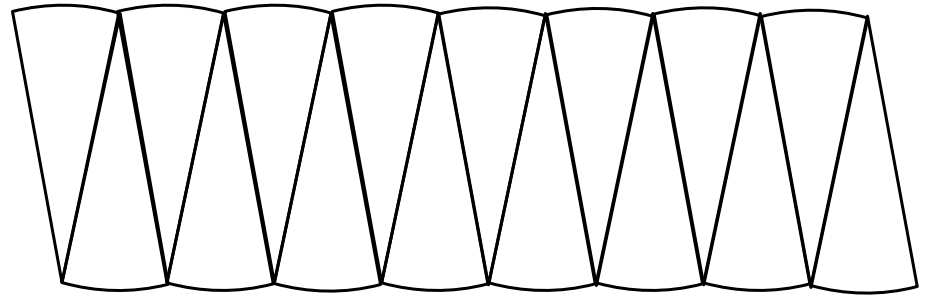
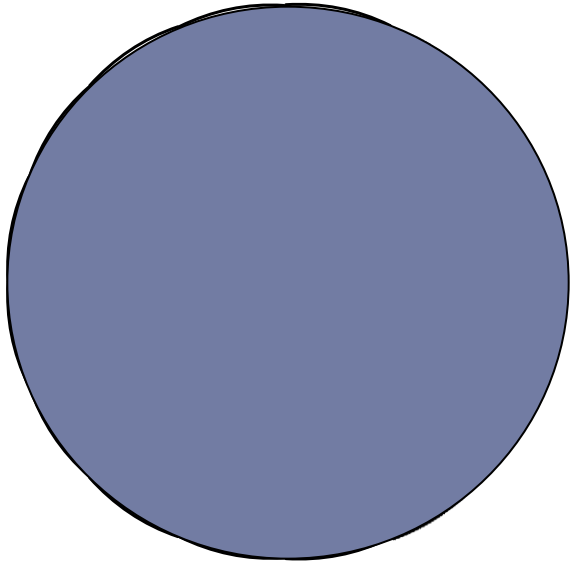




Grade 5

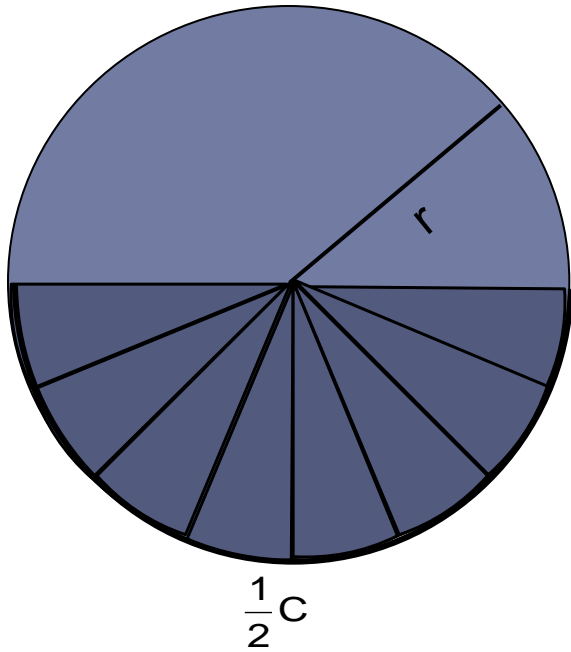






Area of the Circle = Area of the Parallelogram





Area of the Circle = Area of the Parallelogram

Area of Parallelogram = bh

$$\text{Area of Circle} = \frac{1}{2} Cr$$

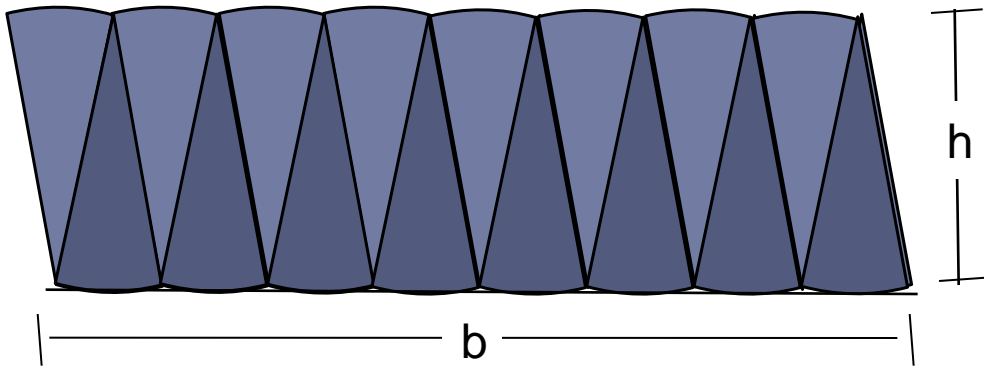
$$= \frac{1}{2} (2\pi r) r$$

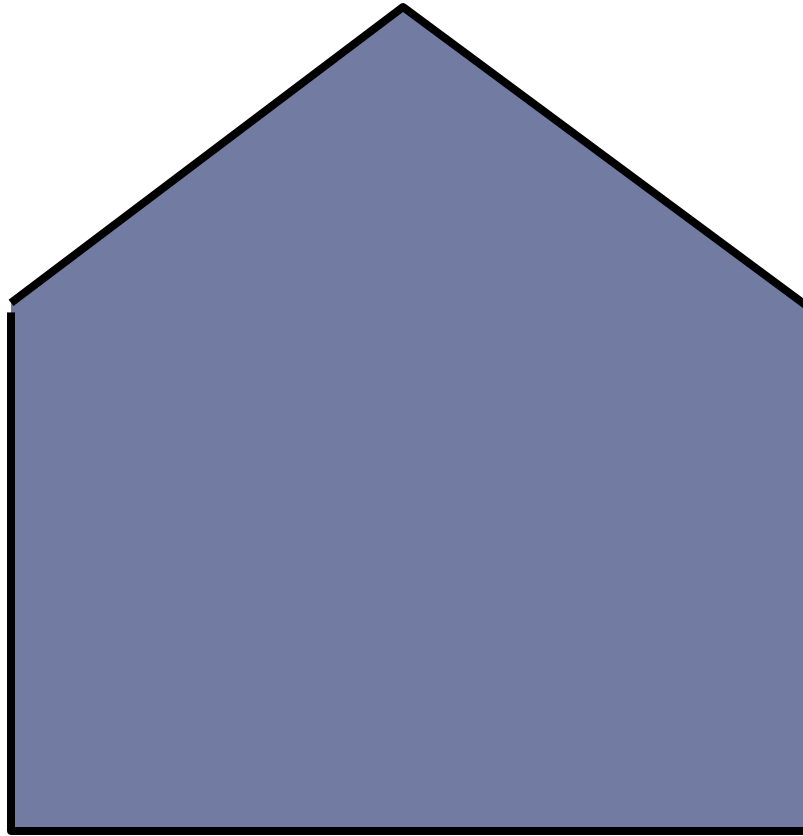
$$= \pi r^2$$

$$b = \frac{1}{2} C$$

$$h = r$$

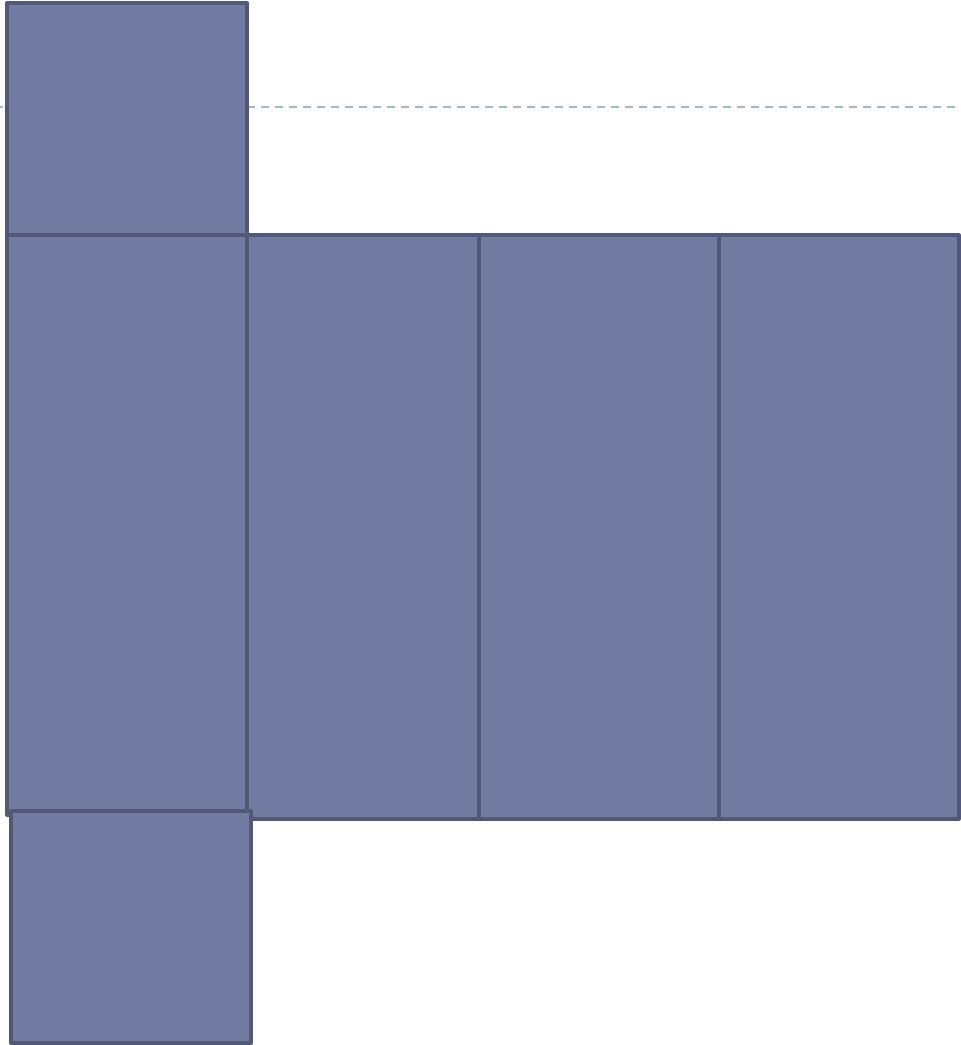
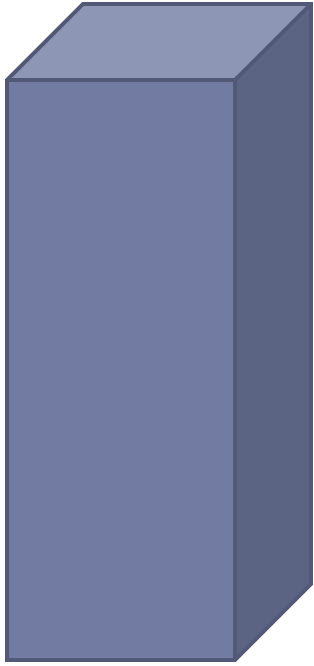
$$C = 2\pi r$$





Grade 6





Patterns and Algebra in the K + 12 Curriculum



▶ Draw the next two figures:

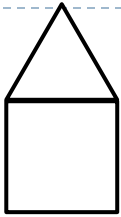


Figure 1

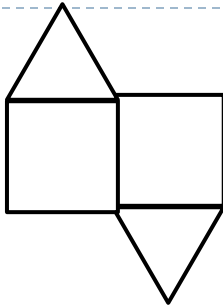


Figure 2

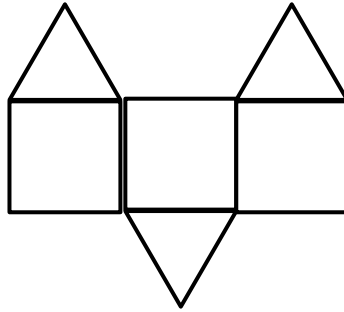


Figure 3

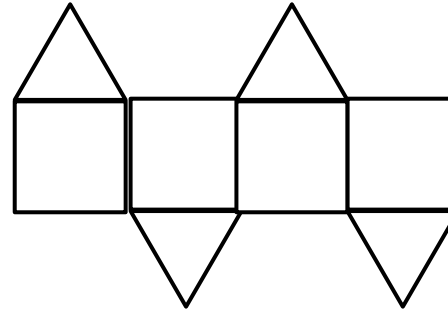


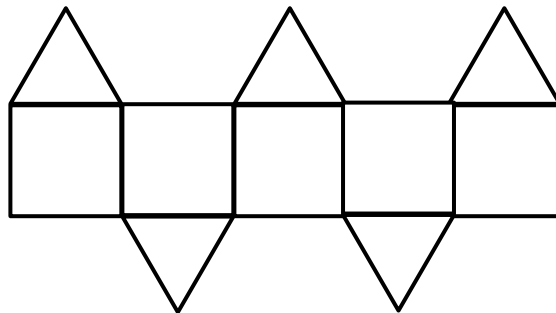
Figure 4

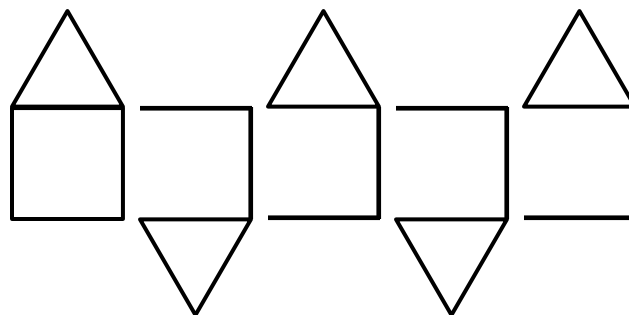
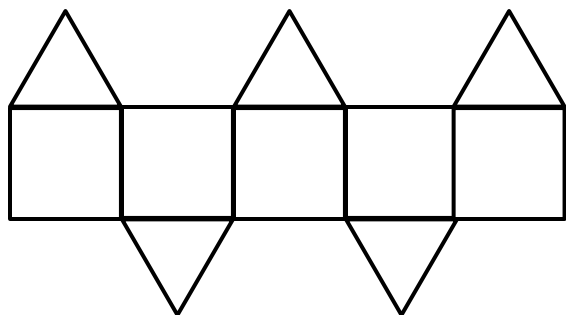
- How many sticks are used in each of Figure 1 to Figure 6?
- How many sticks will there be for Figure 10?



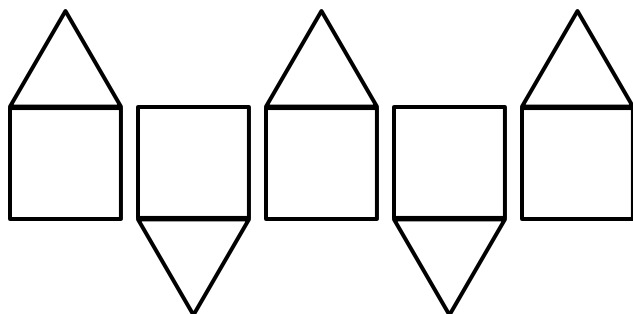
Problem

How many sticks were used in the figure below? Count systematically.

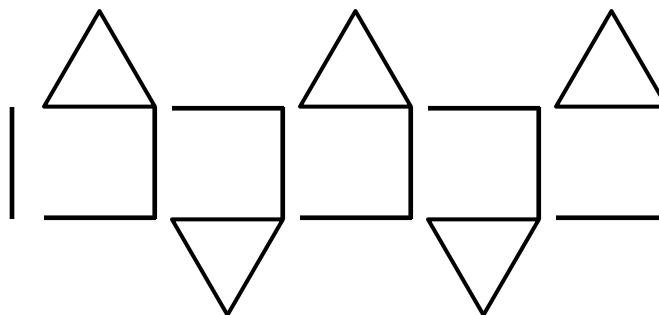




$$6 + (5 - 1) \times 5$$



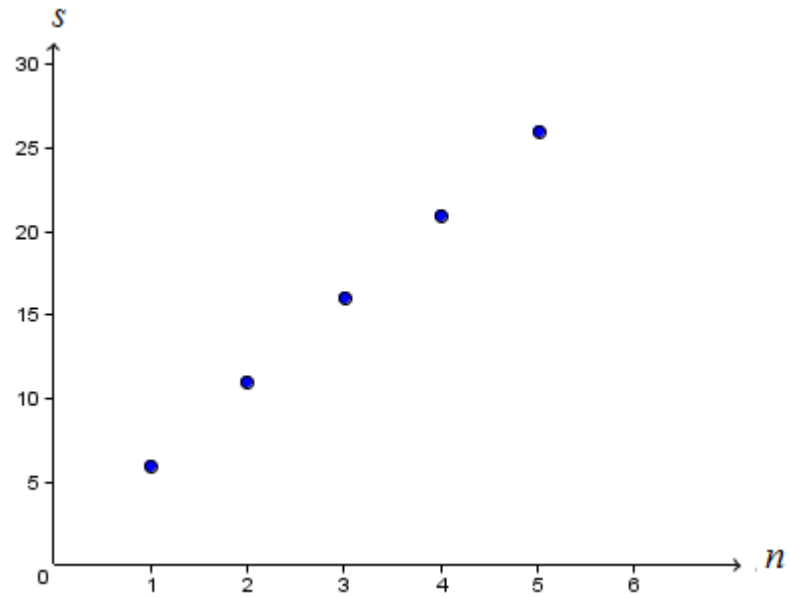
$$(5 \times 6) - (5 - 1)$$



$$1 + (5 \times 5)$$



n	1	2	3	4	5
s	6	11	16	21	26

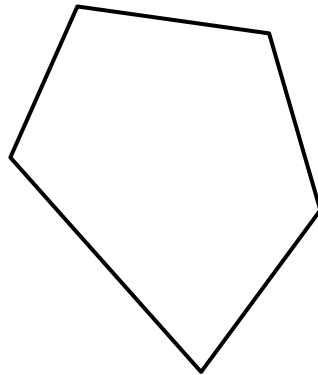


$$s = 1 + 5n$$

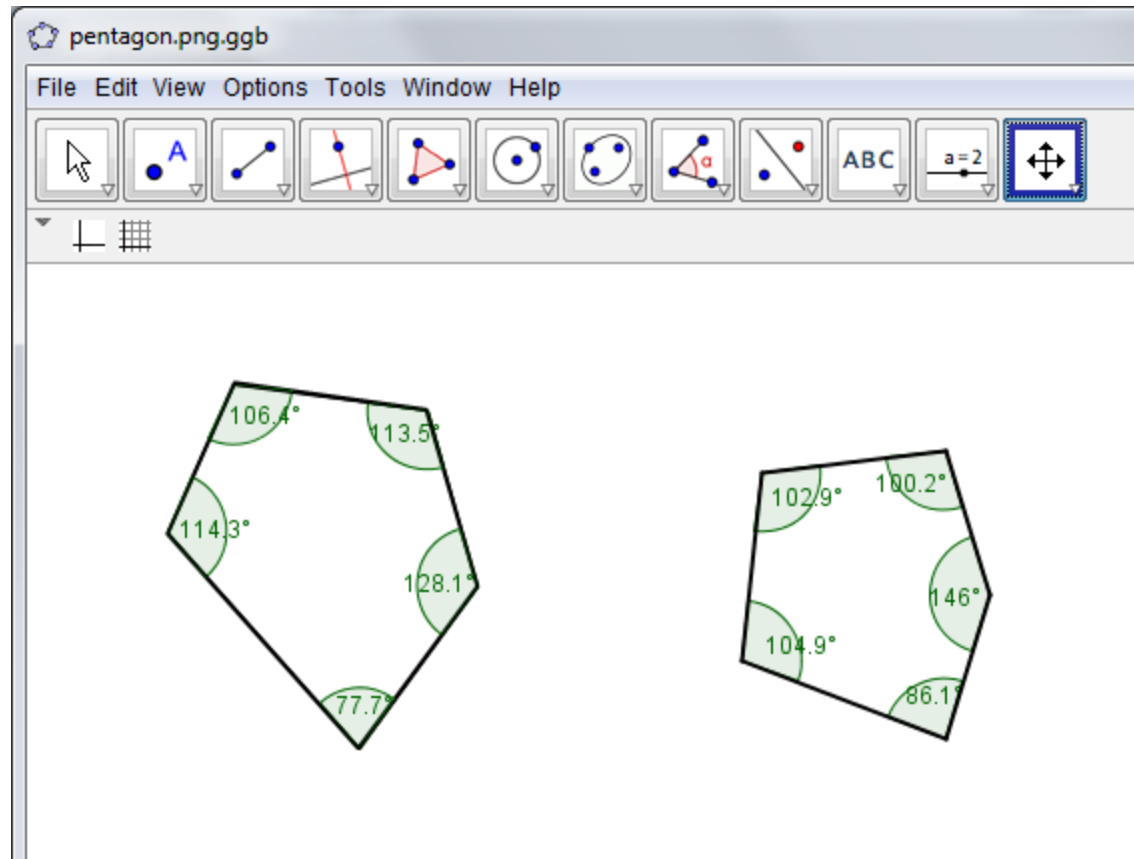


Problem

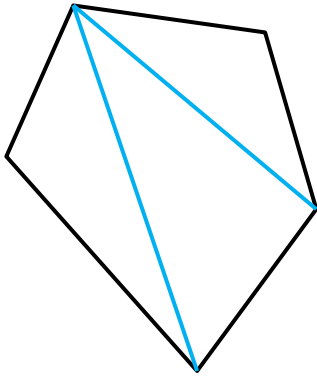
What is the sum of the measures of the interior angles of the pentagon shown?



Solution 1: Using GeoGebra

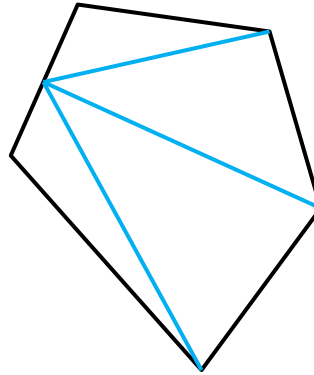


Solution 2



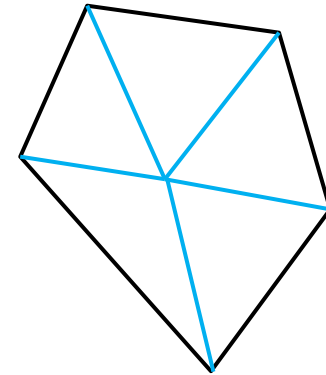
$$a = 180(5-2)$$

Solution 3



$$a = 180(5 - 1) - 180$$

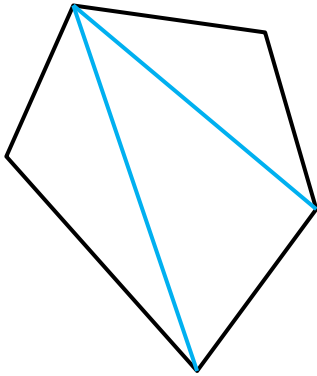
Solution 4



$$a = 180(5) - 360$$

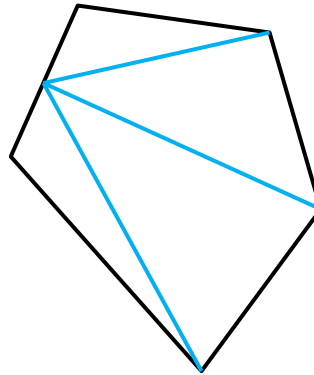


Solution 2



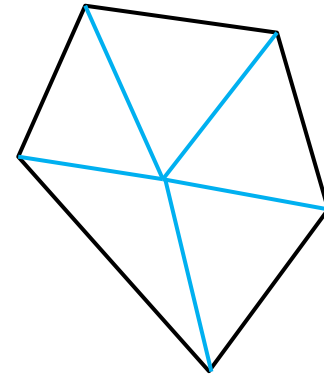
$$a = 180(n-2)$$

Solution 3



$$a = 180(n - 1) - 180$$

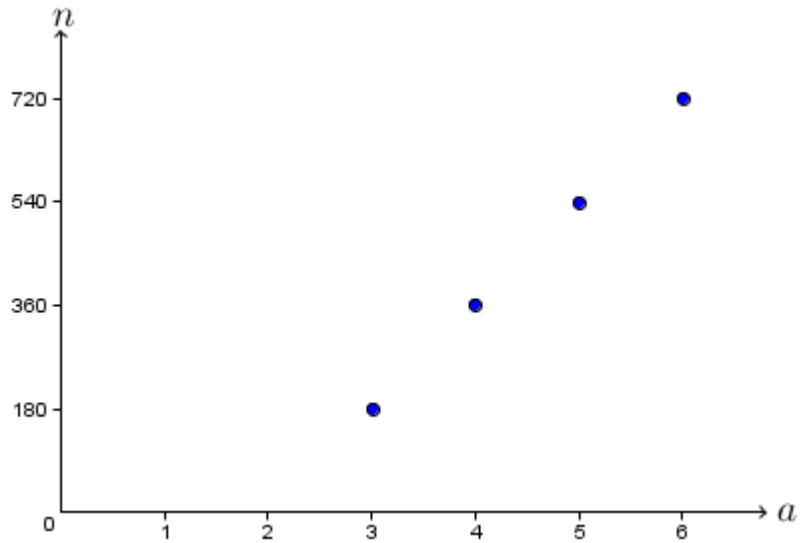
Solution 4



$$a = 180n - 360$$



n	3	4	5	6
a	180°	360°	540°	720°



$$a = 180^\circ(n-2)$$



Problem

In a small gathering, there are 5 guests. If each guest shakes hands with the other guests only once, how many handshakes are there in all?

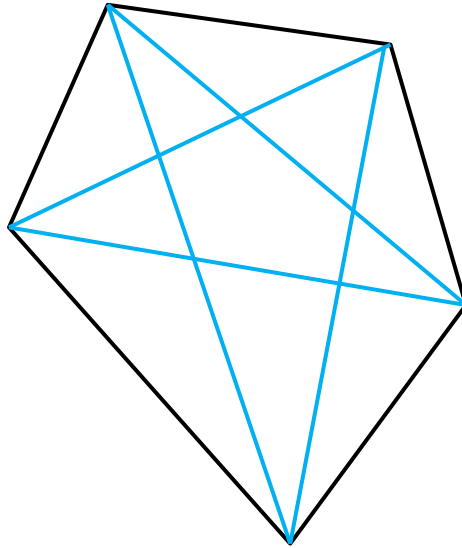


Solution I

	A	B	C	D	E
A	AA	A B	A C	A D	AE
B	AB	BB	BC	BD	BE
C	AC	B C	C C	C D	CE
D	AD	B D	C D	D D	DE
E	AE	BE	CE	DE	E E

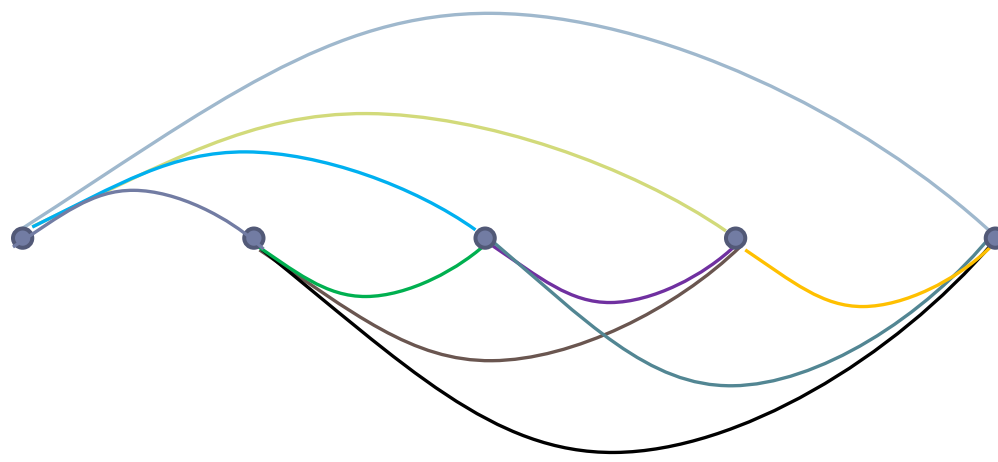
$$\left(\frac{1}{2}\right)[(5 \times 5) - 5]$$





$$\left(\frac{1}{2}\right)(5 \times 4)$$





$$4 + 3 + 2 + 1$$



Thank you!

